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Center projection with and without gauge fixing

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Abstract: We consider projections of SU(2) lattice link variables onto Z_2 center and U(1) subgroups, with and without gauge-fixing. It is shown that in the absence of gauge-fixing, and up to an additive constant, the static quark potential extracted from projected variables agrees *exactly* with the static quark potential taken from the full link variables; this is an extension of recent arguments by Ambjørn and Greensite, and by Ogilvie. Abelian and center dominance is essentially trivial in this case, and seems of no physical relevance. The situation changes drastically upon gauge fixing. In the case of center projection, there are a series of tests one can carry out, to check if vortices identified in the projected configurations are physical objects. All these criteria are satisfied in maximal center gauge, and we show here that they all fail in the absence of gauge fixing. The non-triviality of center projection is due entirely to the maximal center gauge-fixing, which pumps information about the location of extended physical objects into local Z_2 observables.

Keywords: Solitons Monopoles and Instantons, Confinement, Lattice Gauge Field Theories.

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Contents

1. Introduction

There is currently a debate in the lattice gauge theory community regarding which type of gauge field configuration is responsible for the confining force. Of course, confinement mechanisms involving monopoles, center vortices, instantons, and various other types of topological objects have been long discussed in the literature, over a period of decades. In recent years, however, some of these proposals are being subjected to numerical tests. In this connection, it is useful to ask if all of these tests really give us new information, or if, instead, certain results turn out as they do for some very trivial reason.

In this article we will be largely concerned with the center vortex theory, and with the (somewhat vague) concept of "center dominance," and most especially with the ability of center projection to identify physical objects in the vacuum. Much of the discussion, however, applies to abelian dominance and abelian projection as well.

We will begin by showing, in section 2, that in the absence of gauge-fixing, and apart from an additive constant, the potential extracted from center-projected and/or abelian-projected lattices agrees *exactly* with the potential derived from the unprojected lattice; i.e. not only at large distances, but also in the Coulomb regime. If this is what is meant by center or abelian dominance, then it is essentially a triviality in the absence of gauge-fixing, having no obvious relevance to the physics of confinement. This result is an extension of remarks by Ambjørn and one of the authors [1], and of recent work by Ogilvie [2].

The situation is much different when gauge-fixing is imposed. Center projection in maximal center gauge has been used to identify the location of center vortices, and it is asserted that these are physical objects. In ref. [3] we reported the results of a series of numerical tests demonstrating the physical nature of vortices identified in center projection; these include such things as the effect of vortices on large, unprojected Wilson loops, asymptotic scaling of the vortex density, and other properties discussed below. It is a compelling illustration of the importance of maximal center gauge-fixing to simply repeat the numerical tests in the absence of gauge-fixing. What we find, in section 3, is that every one of these tests of "physicality" fails, when no gauge-fixing is employed. This failure is not at all surprising. Vortices are located using local operators (the center-projected plaquettes), and in the absence of a global gauge-fixing these operators can hardly be expected to contain information about infrared physics. But the failures of the no gauge-fixing case serve to highlight the remarkable, and highly non-trivial, fact that each test is satisfied when maximal center gauge is imposed.

Finally, in section 4, we note that the "center dominances" which are obtained with and without gauge-fixing are not really the same. Without gauge-fixing, projected and unprojected potentials are identical, starting out Coulombic at short distances and going linear at large distances. Imposing maximal center gauge, the center-projected potential is nearly linear everywhere, from one lattice spacing onwards. We explain why this "precocious linearity" is to be expected, if projected plaquettes locate the genuine confining configurations (center vortices) in the unprojected lattice. Section 5 contains some concluding remarks.

2. Center dominance without gauge fixing

Let $U(C)$ denote the product of link variables around loop C in SU(2) lattice gauge theory, and let $U(R,T)$ in particular denote the link product around a rectangular $R \times T$ loop. Suppose, instead of computing the potential in the usual way, i.e.

$$V(R) = \lim_{T\to\infty} -\log\left[\frac{\langle \mathrm{Tr}[U(R,T+1)]\rangle}{\langle \mathrm{Tr}[U(R,T)]\rangle}\right], \tag{2.1}$$

we calculate the potential from only the sign of the Wilson loop

$$V_S(R) = \lim_{T\to\infty} -\log\left[\frac{\langle \mathrm{sign}\mathrm{Tr}[U(R,T+1)]\rangle}{\langle \mathrm{sign}\mathrm{Tr}[U(R,T)]\rangle}\right]. \tag{2.2}$$

This was done done numerically in ref. [4] (although of course without taking T to ∞), with the surprising result that the sign-projection potential $V_S(R)$ and the full potential $V(R)$ agree, except for the very smallest loops. However, this agreement can actually be explained in a simple way, as shown in ref. [1] (see also [5, 6]).

We first need the result that for large T,

$$W_{1/2}[R,T] \gg W_{3/2}[R,T] \gg W_{5/2}[R,T] \gg \cdots \tag{2.3}$$

where

$$W_j[C] = \frac{1}{2j+1}\langle \chi_j[U(C)]\rangle \tag{2.4}$$

and $\chi_j[g]$ is the SU(2) group character in representation j. The above inequality (2.3) can be seen as follows. In all cases, we consider very large T, and j =half-integer. Begin with R in the Coulombic regime. The leading contribution, just coming from 1-gluon exchange, is

$$W_j[R,T] \approx \exp\left[-T\left(-\frac{g^2_{eff}(R)C_j}{4\pi R} + 2bg^2_{eff}(a)C_j\right)\right], \tag{2.5}$$

where $C_j = j(j+1)$ is the quadratic Casimir and b is a constant of $O(1)$.[1] The first term is the Coulomb contribution, the second term is the self-energy, and we have neglected terms subleading in T. Because C_j increases with j, and because the self-energy exceeds the Coulomb term, the inequality (2.3) at large T follows. As R is increased, and the loop probes forces in the Casimir-scaling regime, the leading loop behavior becomes

$$W_j[R,T] \approx \exp\left[-T\left(\sigma_j R + 2bg^2_{eff}(a)C_j\right)\right], \tag{2.6}$$

where the string tension σ_j increases with j. Casimir scaling behavior is deduced from numerical results; c.f. [8]. Here again, since σ_j and C_j increase with j, the inequality (2.3) is satisfied. Finally, in the asymptotic regime, the color charges of the half-integer representations are screened via binding to gluons down to $j=\frac{1}{2}$, and we have

$$W_j[R,T] \approx \exp\left[-T\left(\sigma_{1/2}R + d_j\right)\right]. \tag{2.7}$$

Color screening of static souces by matter in the fundamental representation ("string-breaking") has only recently been observed in Monte Carlo simulations [9]. The screening of higher representation static sources by gluons is also expected to occur, on the basis of very general energetics arguments, at sufficiently large separations; this screening can be easily demonstated in the context of the strong coupling expansion. The term d_j will have contributions from two sources. The first, for $j > \frac{1}{2}$, is the bound state energy of gluons required to screen the heavy quark color charge to $j = 1/2$. The higher j is, the more gluons are required to screen the charge, and the larger the energy of the "gluelump".[2] A second contribution to d_j comes from

[1] This result is obtained by standard perturbative evaluation of the Wilson loop; see e.g. [7].

[2] An increase in the mass of the gluelump with increasing numbers of bound gluons seems rather likely, although the gluelump masses associated with higher representation sources (beyond adjoint) have not yet been calculated numerically, and we do not attempt to do so here.

the heavy-quark perturbative self-energy contribution, proportional to g^2C_j. Both contributions cause d_j to increase with j, and again (2.3) is obtained. The conclusion is that, for any R, and any two half-integer representations $j_1 > j_2$,

$$\lim_{T\to\infty} \frac{W_{j_1}[R,T]}{W_{j_2}[R,T]} = 0\,. \tag{2.8}$$

We now make the character expansion

$$\text{signTr}[g] = \sum_{j=\frac{1}{2},\frac{3}{2},\frac{5}{2},\ldots} a_j \chi_j[g]\,, \tag{2.9}$$

with

$$\begin{aligned} a_{1/2} &= \int dg\ \text{signTr}[g]\chi_{1/2}[g] \\ &= \frac{8}{3\pi}\,. \end{aligned} \tag{2.10}$$

Then, taking into account (2.8), this means that for large T

$$\langle \text{signTr}[U(R,T)]\rangle \approx \frac{8}{3\pi} \langle \text{Tr}[U(R,T)]\rangle \tag{2.11}$$

and the equality of the full potential $V(R)$ and projected potential $V_S(R)$ follows immediately. From its derivation, which simply follows from the character expansion and eq. (2.3), it is not clear to us that the equality of $V_S(R)$ and $V(R)$ bears directly on the confinement issue. Note that this equality holds even in the Coulombic regime, before confinement physics comes into play.

A closely related observation has been made by Ogilvie [2], this time concerning projections of link variables, rather than loop variables. Consider a projection $U_\mu(x) \to H_\mu(x)$, of SU(N) link variables onto some subgroup H of SU(N). Then it is shown in ref. [2] that, in the absence of gauge-fixing, the asymptotic string tension extracted from the projected link variables in Tr$H(C)$ agrees with the asymptotic string tension derived from the full link variables.

In fact, for both abelian and center projection, the statement concerning potentials from full and projected link variables can be made very much stronger than the result stated in [2]. As in ref. [2], we consider such projections in the absence of any associated gauge-fixing. We begin with center projection. The Wilson loop on a center-projected lattice is defined to be

$$\begin{aligned} W_P(C) &\equiv \left\langle \prod_{l\in C} \text{signTr}[U_l] \right\rangle \\ &= \frac{1}{Z}\int DU \prod_{l\in C} \sum_{j_l=\frac{1}{2},\frac{3}{2},\frac{5}{2},\ldots} a_{j_l}\chi_{j_l}[U_l] e^{-S}\,. \end{aligned} \tag{2.12}$$

Let (x_l, y_l) denote the (path-ordered) endpoints of link $l \in C$, with the convention $U_l = U_\mu(x_l)$ if $y_l = x_l + \hat{\mu}$, and $U_l = U^\dagger_\mu(y_l)$ if $x_l = y_l + \hat{\mu}$. Applying the familiar trick of inserting an integration over gauge transformations $1 = \int Dg$ followed by a change of variables $U \to gUg^\dagger$,

$$\begin{aligned} W_P(C) &= \frac{1}{Z}\int DUDg \prod_{l\in C}\left(\sum_{j_l=\frac{1}{2},\frac{3}{2},\frac{5}{2},\dots} a_{j_l}\chi_{j_l}\left[g(x_l)U_l g^\dagger(y_l)\right]\right) e^{-S} \\ &= \frac{1}{Z}\int DU \sum_{j=\frac{1}{2},\frac{3}{2},\frac{5}{2},\dots} a_j^{P(C)}\frac{1}{d_j^{P(C)-1}}\chi_j[U(C)]e^{-S} \\ &= \sum_{j=\frac{1}{2},\frac{3}{2},\frac{5}{2},\dots} a_j^{P(C)}\frac{1}{d_j^{P(C)-2}} W_j(C)\,, \end{aligned} \tag{2.13}$$

where $P(C)$ is the loop perimeter, and we have used the identity

$$\int dg \chi_j[Ug^\dagger]\chi_k[gU'] = \frac{1}{d_j}\chi_j[UU']\delta_{jk}\,, \tag{2.14}$$

with $d_j = 2j+1$. Finally, making use of (2.8) we have

$$W_P(R,T) \overset{T\to\infty}{\Longrightarrow} 4\left(\frac{4}{3\pi}\right)^{P(C)} W_{1/2}(R,T) \qquad \textbf{(center projection)} \tag{2.15}$$

and the equality of center-projected and full potentials up to an additive constant

$$V_P(R) + 2\ln\left(\frac{4}{3\pi}\right) = V(R) \tag{2.16}$$

again follows immediately.

Abelian links A are obtained from full link variables U

$$U = \begin{pmatrix} \cos\phi\; e^{i\theta} & \sin\phi\; e^{i\chi} \\ -\sin\phi\; e^{-i\chi} & \cos\phi\; e^{-i\theta}\end{pmatrix} \tag{2.17}$$

by setting

$$A(U) = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta}\end{pmatrix} \tag{2.18}$$

and the abelian Wilson loop is defined as

$$W_P[C] = \frac{1}{2}\left\langle \mathrm{Tr}[\prod_{l\in C} A(U_l)]\right\rangle . \tag{2.19}$$

Abelian links, unlike center projected links, are not class functions, and cannot be expanded in SU(2) group characters. Instead we make use of the fact that, in the absence of gauge-fixing, only the gauge-invariant component O_{inv} of an operator O will contribute to the expectation value of O, where O_{inv} is given by

$$O_{inv}[\{U_l\}] = \int Dg\; O\left[\{g(x_l)U_l g^\dagger(y_l)\}\right]. \tag{2.20}$$

This fact is readily verified by again making the insertion $1 = \int Dg$ and change of variables $U \to gUg^\dagger$ in the functional integral, which gives, in the present case,

$$W_P[C] = \frac{1}{Z}\int DU \left\{ \int Dg \, \frac{1}{2}\mathrm{Tr}\left[\prod_{l\in C} A\left(g(x_l)U_l g^\dagger(y_l)\right)\right]\right\} e^{-S} \tag{2.21}$$

The quantity in braces is gauge invariant, and can now be expanded in SU(2) group characters

$$\int Dg \, \frac{1}{2}\mathrm{Tr}\left[\prod_{l\in C} A\left(g(x_l)U_l g^\dagger(y_l)\right)\right] = \sum_j a_j \chi_j \left[\prod_{l\in C} U_l\right] \tag{2.22}$$

where

$$\begin{aligned} a_j &= \int \prod_{l\in C} dU_l \int Dg \, \frac{1}{2}\mathrm{Tr}\left[\prod_{l\in C} A\left(g(x_l)U_l g^\dagger(y_l)\right)\right] \chi_j \left[\prod_{l\in C} U_l\right] \\ &= \int \prod_{l\in C} dU_l \, \frac{1}{2}\mathrm{Tr}\left[\prod_{l\in C} A(U_l)\right] \chi_j \left[\prod_{l\in C} U_l\right] \end{aligned} \tag{2.23}$$

and in particular

$$\begin{aligned} a_{1/2} &= \left\{ \prod_{l\in C} \frac{1}{2\pi^2}\int_0^{\pi/2} d\phi_l \cos\phi_l \sin\phi_l \int_{-\pi}^{\pi} d\theta_l \int_{-\pi}^{\pi} d\chi_l \right\} \frac{1}{2}\mathrm{Tr} \prod_{l'\in C} \begin{pmatrix} e^{i\theta_{l'}} & 0 \\ 0 & e^{-i\theta_{l'}} \end{pmatrix} \times \\ &\quad \times \mathrm{Tr} \prod_{l''\in C} \begin{pmatrix} \cos\phi_{l''} e^{i\theta_{l''}} & \sin\phi_{l''} e^{i\chi_{l''}} \\ -\sin\phi_{l''} e^{-i\chi_{l''}} & \cos\phi_{l''} e^{-i\theta_{l''}} \end{pmatrix} \\ &= \left(\frac{2}{3}\right)^{P(C)} \end{aligned} \tag{2.24}$$

Thus we can again express

$$W_P[C] = \sum_{j=\frac{1}{2},\frac{3}{2},\frac{5}{2},\ldots} a_j \left\langle \chi_j \left[\prod_{l\in C} U_l\right]\right\rangle \tag{2.25}$$

For the same reasons as before, the higher representations can be neglected as $T \to \infty$, so that

$$W_P(R,T) \overset{T\to\infty}{\Longrightarrow} 2\left(\frac{2}{3}\right)^{P(C)} W_{1/2}(R,T) \qquad \textbf{(abelian projection)} \tag{2.26}$$

from which the exact equality (up to an additive constant) of the abelian-projected and full potentials follows. It is interesting that, although the expressions (2.15) and (2.26) relating $W_P(R,T)$ to $W_{1/2}(R,T)$ only hold exactly as $T \to \infty$, we have found in numerical simulations that these are in fact very good approximations to even the smallest projected 1×1 and 1×2 loops at, e.g., $\beta = 2.3$.

The initial interest in abelian projection (and, more recently, center projection) was sparked by the discovery that the string tension of abelian projected lattices scales, and agrees (at least approximately) with the full string tension [10]. The triviality of this result, in the absence of gauge fixing, suggests that no strong conclusions can be drawn from abelian and/or center dominance alone. However, results obtained from lattice projection do not end with abelian or center dominance. In particular, in the case of center projection in maximal center gauge, we claim to be able to identify the location of confining center vortices in the unprojected lattice and to show, via a series of tests, that these are physical objects rather than artifacts of the projection. The existence of center dominance (2.16) in the absence of gauge-fixing then raises a natural question: Are the tests for the physical nature of vortices *also* somehow trivial? Suppose one tries to identify center vortices without any gauge fixing, and repeats the same series of tests. What happens?

3. Can one find vortices without gauge-fixing?

The "direct" version of maximal center gauge, used in most of our work, is defined as the gauge which maximizes

$$\sum_{x,\mu} \Big|\mathrm{Tr}[U_\mu(x)]\Big|^2 . \tag{3.1}$$

There is also an "indirect" version, which begins from maximal abelian gauge and then uses the remnant $U(1)$ symmetry to maximize an expression like (3.1), with the full link variables replaced by the abelian projected links. Center projection (in SU(2) gauge theory) is a mapping of the SU(2) lattice link variables U_μ to Z_2 link variables Z_μ via

$$Z_\mu(x) \equiv \mathrm{signTr}[U_\mu(x)] . \tag{3.2}$$

The excitations of a Z_2 lattice are Z_2 vortices, which are line-like in D=3 dimensions, and surface-like in D=4 dimensions. We refer to the vortices on the center-projected lattice as projected vortices, or just "P-vortices". A plaquette on the original lattice is said to be "pierced" by a P-vortex if the corresponding plaquette on the projected lattice has the value -1. We define vortex-limited Wilson loops $W_n(C)$ to be Wilson loops evaluated on the original, unprojected lattice, with the following "cut" in the Monte Carlo data: $W_n(C)$ is only evaluated for those loops C in which exactly n plaquettes in the minimal area are pierced by P-vortices. In the same way, $W_{even}(C)$ and $W_{odd}(C)$ refer to loops pierced by even and odd numbers of P-vortices, respectively, and one can also define Creutz ratios $\chi_n(I,J)$, $\chi_{even}(I,J)$ extracted from the W_n and W_{even}.

We now list our reasons, which are the outcome of a series of tests, for believing that "thin" P-vortices in the center-projected lattice locate "thick" center vortices in the unprojected lattice, and that these thick vortices are physical objects:

1. **P-Vortices locate center vortices.** Vortex excitations in the center-projected configurations, in direct maximal center gauge, locate center vortices in the full, unprojected lattice. The evidence for this comes from the fact that

$$W_n(C)/W_0(C) \to (-1)^n \qquad \text{and} \qquad W_{odd}(C) \to -W_{even}(C) \tag{3.3}$$

as loop area increases.

2. **No vortices ⇒ no confinement.** When Wilson loops in SU(2) gauge theory are evaluated in subensembles of configurations with no vortices (or only an even number of vortices) piercing the loop, the string tension disappears; i.e.

$$\chi_0(I,J) \to 0\,, \qquad \chi_{even}(I,J) \to 0 \tag{3.4}$$

as loop area increases.

3. **Vortex density scales.** The variation of P-vortex density with coupling β goes exactly as expected for a physical quantity with dimensions of inverse area [11, 3].

4. **P-vortices are strongly correlated with action density.** Plaquettes which are pierced by P-vortices have a very much higher *unprojected* plaquette action than the vacuum average. Monopole loops lie on P-vortices [12].

Regarding this last point, it is also found that monopoles, identified in the maximal abelian gauge, lie along center vortices, found in the indirect maximal center gauge, in a monopole-antimonopole chain. The non-abelian field strength of monopole cubes, above the lattice average, is directed almost entirely along the associated center vortices. Monopoles themselves appear to be rather undistinguished regions of vortices. In fact, it is natural to interpret them as simply "kinks" of the vortex magnetic field in maximal abelian gauge, as explained in ref. [12], where the magnetic field components in the vortex direction reverse their orientation in color space (e.g. from the $+\sigma_3$ to the $-\sigma_3$ direction).

Finally, there is

5. **Precocious Linearity.** There is no Coulomb potential on the center-projected lattice at short distances. The projected potential is linear from the beginning, with a string tension in agreement with the one extracted asymptotically on the unprojected lattice.

As far as the "precocious linearity" of the projected potential is concerned, we know already, from the validity of eq. (2.16) at all distances, that this result is not obtained in the absence of gauge fixing. A discussion of this point will be deferred to the next section. We will now present data on the other criteria, taken from Monte

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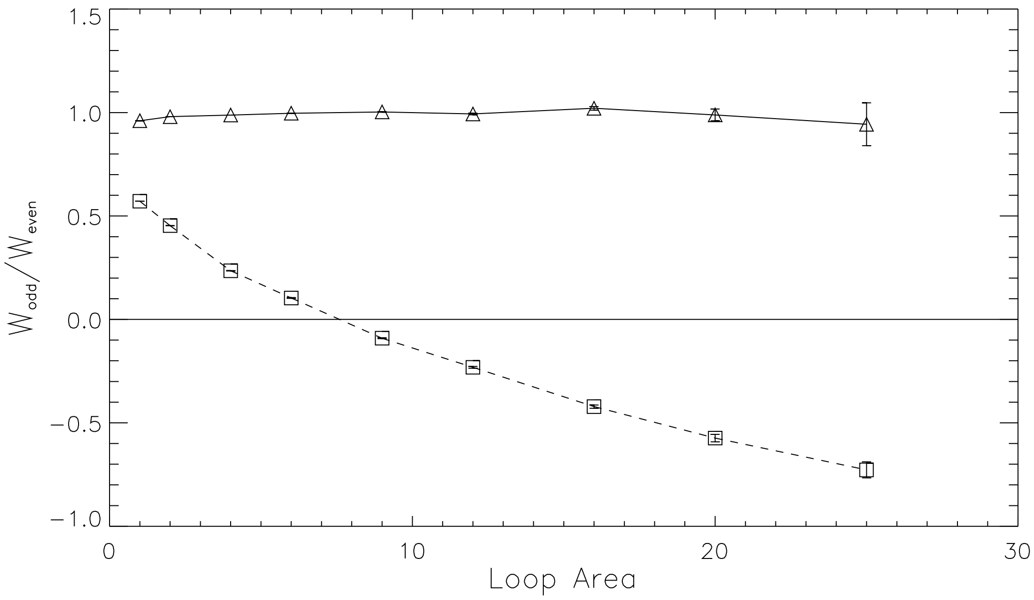

Figure 1: Ratio of the Odd-Vortex to the Even-Vortex Wilson loops, $W_{odd}(C)/W_{even}(C)$, vs. loop area at $\beta = 2.3$, with (squares) and without (triangles) maximal center gauge-fixing.

Carlo simulations with no gauge fixing, and compare it to our previous results, in which the maximal center gauge was imposed.

The first check is whether P-vortices still locate center vortices, in the absence of maximal center gauge fixing. The criterion (c.f. ref. [3]) is that $W_n(C)/W_0(C) \to (-1)^n$; however, since the density of P-vortices is quite large in the absence of gauge fixing, it is hard to get good statistics for $W_0(C)$ for the larger loops. Instead, we check the ratio W_{odd}/W_{even}. If P-vortices locate center vortices, then we should also see

$$\frac{W_{odd}(C)}{W_{even}(C)} \to -1 \tag{3.5}$$

as the loops get large. The data at $\beta = 2.3$ on a 14^4 lattice is shown in fig. 1. The ratios obtained under maximal center gauge are tending towards -1 as the loop area increases, which indicates that P-vortices do indeed locate center vortices (we have shown W_1/W_0 and W_2/W_0 data elsewhere [3], this is also consistent with P-vortices locating center vortices). In obvious contrast, the ratio obtained without gauge-fixing is consistent with $+1$, and has little variation with loop size. There is no reason, in this case, to suppose that P-vortices locate center vortices. So this is the first test to fail in the no gauge-fix case.

The second test is to see if the no-vortex or even-vortex Wilson loops lose their string tensions, because of the zero- or even-vortex restrictions. Again, because of statistics, we can only look at the even-vortex loops in the no gauge-fix case. In fig. 2 we see that in maximal center gauge the Creutz ratios $\chi_{even}(I, I)$ do indeed

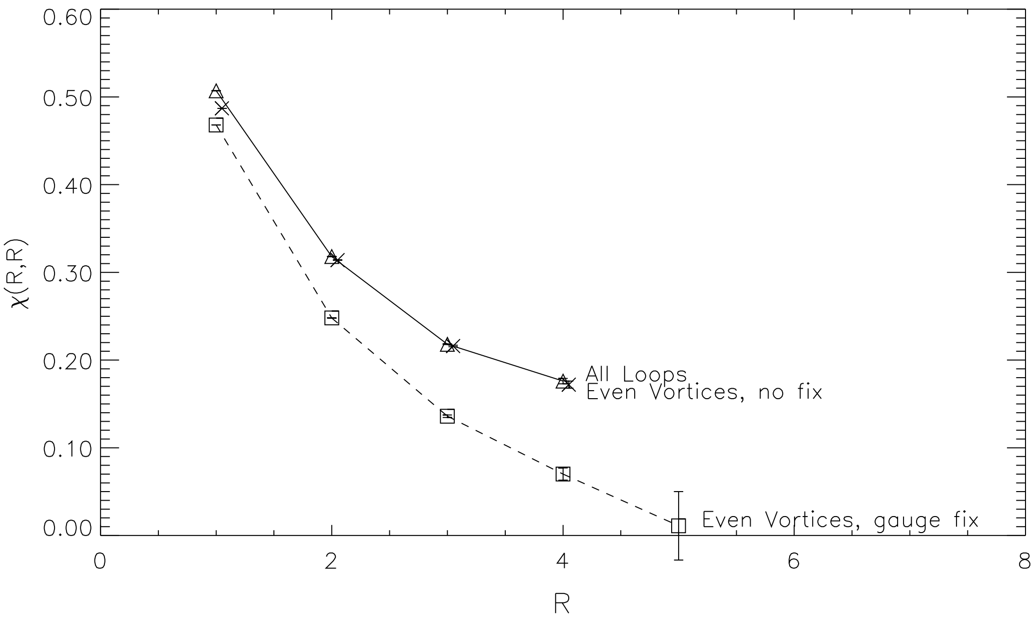

Figure 2: Creutz ratios $\chi_{even}(R, R)$ extracted from loops pierced by even numbers of P-vortices, defined with (squares) and without (crosses) maximal center gauge fixing, as compared to the usual Creutz ratios $\chi(R, R)$ (triangles) at $\beta = 2.3$. Data points for $\chi_{even}(R, R)$ with no gauge-fixing (crosses) have been slightly displaced in R, to help distinguish them from the $\chi(R, R)$ data points (triangles).

drop to zero as loop area increases, while in the absence of gauge fixing there is no discernable difference between $\chi_{even}(I, I)$ and the usual Creutz ratios $\chi(I, I)$.

Of course, the results shown in figs. 1 and 2, for the no gauge-fixing case, are closely related. From fig. 1 we have that $W_{even}(C) \approx W_{odd}(C)$ in the no gauge-fix case, which implies $W(C) \approx W_{even}(C)$. The equality of Creutz ratios $\chi_{even}(I, I) = \chi(I, I)$ follows, as seen in fig. 2.

The failure of these two tests, in the case of no gauge-fixing, could also have been anticipated analytically. This will be shown in an appendix.

For the test of asymptotic scaling, we first define p to be the fraction, and N_{vor} to be the total number, of center projected plaquettes with value -1. N_{vor} is also the total area of all P-vortices on the dual lattice, and we denote by N_T the total number of all plaquettes on the lattice. Then

$$\begin{aligned} p &= \frac{N_{vor}}{N_T} = \frac{N_{vor} a^2}{N_T a^4} a^2 \\ &= \frac{\text{Total Vortex Area}}{6 \times \text{Total Volume}} a^2 \\ &= \frac{1}{6} \rho a^2 \, . \end{aligned} \tag{3.6}$$

If ρ, which is the area of P-vortices per unit volume, is a fixed quantity in physical units, then according to asymptotic freedom we should find, in the scaling regime,

$$p = \frac{1}{6}\frac{\rho}{\Lambda^2}\left(\frac{6\pi^2}{11}\beta\right)^{102/121}\exp\left[-\frac{6\pi^2}{11}\beta\right], \tag{3.7}$$

where a is the lattice spacing. The fraction p is related to the 1-plaquette term in center projection

$$W_{cp}(1,1) = (1-p) + p \times (-1) = 1 - 2p\,. \tag{3.8}$$

A plot of the P-vortex density p versus coupling β, as shown in fig. 3. The straight line is the asymptotic freedom expression (eq. (3.7)), with the choice $\sqrt{\rho/(6\Lambda^2)} = 50$. For the data obtained in maximal center gauge, the scaling of P-vortex densities seems really compelling (a result which was first reported for the "indirect" version of this gauge in ref. [11]). By contrast, the density of P-vortices obtained in the absence of gauge-fixing shows no sign at all of asymptotic scaling.

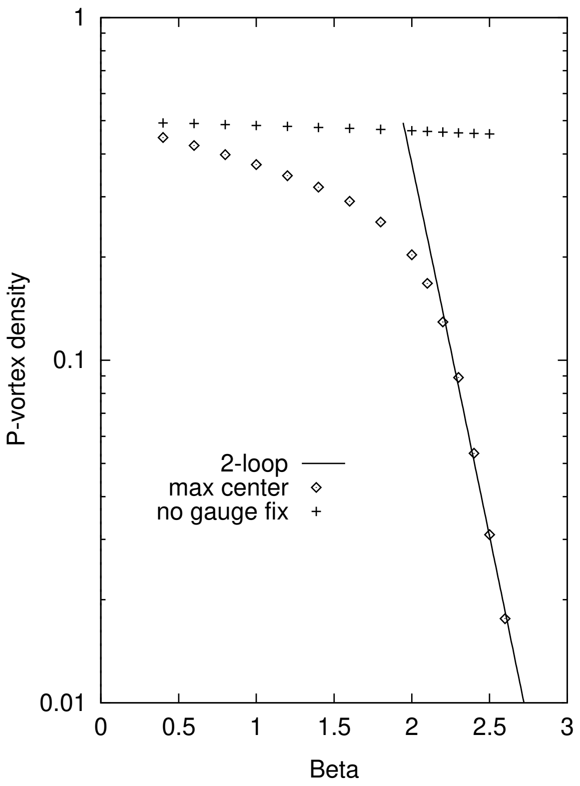

Figure 3: Evidence for asymptotic scaling of the P-vortex density, defined as the fraction p of plaquettes pierced by P-vortices (one-sixth the average area occupied by P-vortices per unit lattice volume). The solid line is the asymptotic freedom prediction of eq. (3.7), with constant $\sqrt{\rho/(6\Lambda^2)} = 50$. Data points with and without maximal center gauge-fixing are shown.

Finally, we can show in fig. 4 a plot of the one-vortex plaquette action $W_1(1,1)$ as a function of β, both for maximal center gauge and no gauge-fixing, compared to the usual plaquette action. For no gauge-fixing, the deviation of vortex plaquettes from average plaquettes is very small. In the maximal center gauge there is a very substantial deviation.

We conclude that the P-vortices identified in maximal center gauge are true physical objects, that can be identified with center vortices. In the absence of gauge fixing there is no indication, apart from a very slight correlation with action density, that individual P-vortex plaquettes identify the location of any physical object.

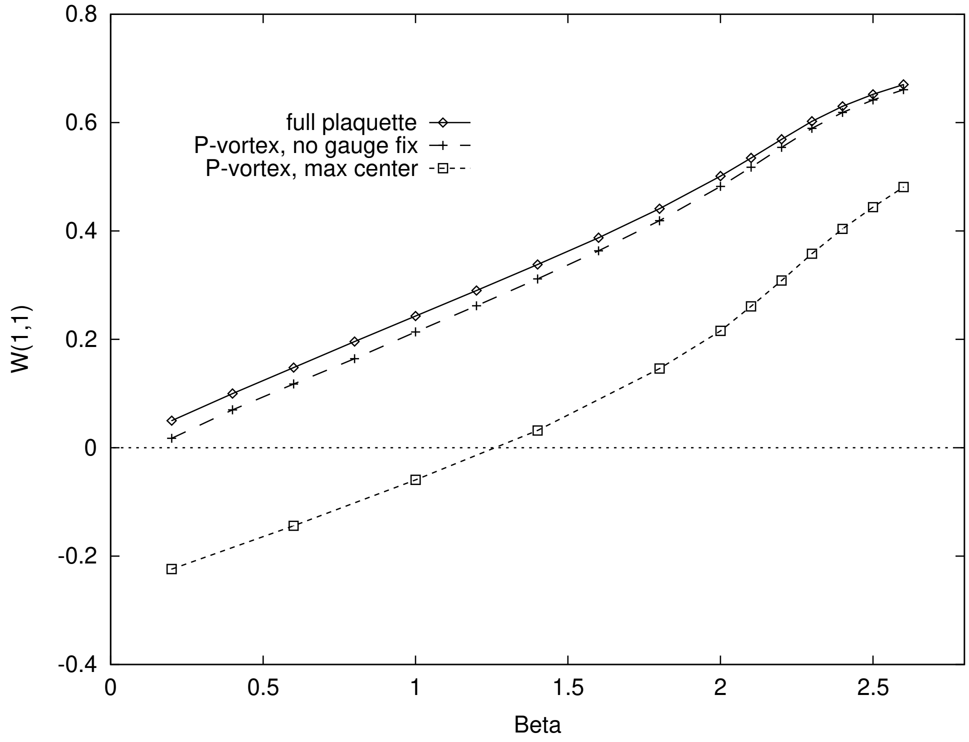

Figure 4: One-plaquette loops $W_1(1,1)$ for plaquettes pierced by P-vortices, evaluated both in maximal center gauge (squares) and with no gauge-fixing (crosses), as compared to the usual one-plaquette loop $W(1,1)$ (diamonds).

4. Precocious linearity

Only the gauge-invariant component O_{inv} of an observable O, obtained by averaging over gauge transformations, contributes to the expectation value $\langle O \rangle$, when the expectation value is computed in the absence of gauge-fixing. For abelian or center-projected Wilson loops, the leading gauge-invariant contribution is the standard, fundamental-representation Wilson loop, which in fact gives the entire contribution to the expectation value in the $T \to \infty$ limit. This is the quick explanation for why projected potentials agree with the full potential in the absence of gauge-fixing. There is, however, no reason for such an agreement to persist when a global gauge-fixing is imposed, and in fact the exact agreement does not persist in general.

In the first place, there is known to be a discrepancy between the string tensions of abelian projected loops obtained in maximal abelian gauge, and string tensions of unprojected loops. A careful study in lattice SU(2) gauge theory at a particular coupling ($\beta = 2.5115$) shows that the string tension of projected loops in maximal abelian gauge is only 92%, rather than 100% of the usual string tension [13]. There is also a difference between string tensions obtained in the "direct" [3] and "indirect" [14] versions of maximal center gauge, with $\sqrt{\sigma}/\Lambda$ differing by $\approx 13\%$ in the two cases. The projected string tensions in the direct version of maximal center

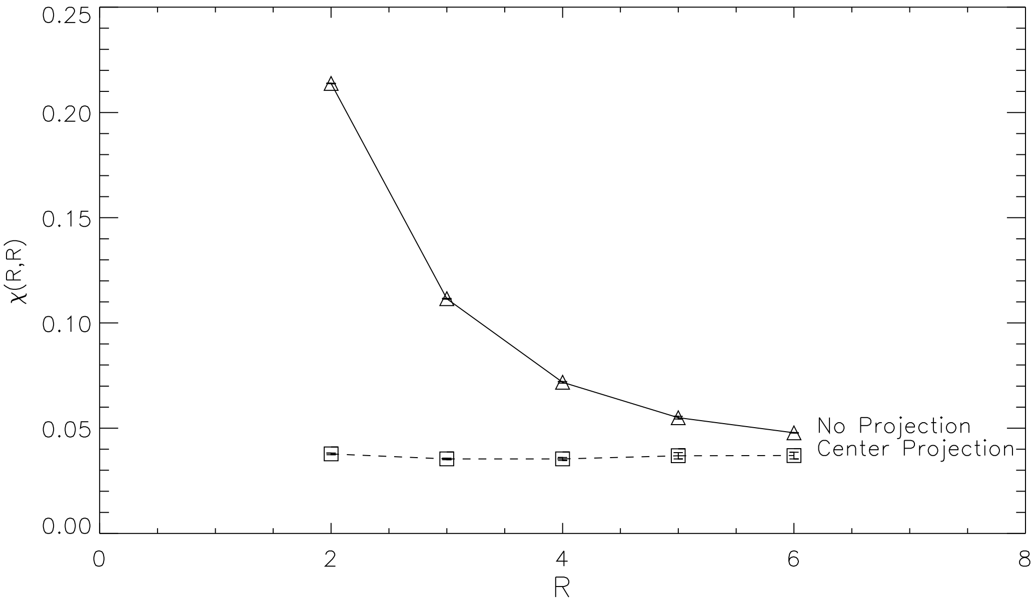

Figure 5: Center-projected Creutz ratios $\chi_{proj}(R,R)$ in maximal center gauge, as compared to the usual Creutz ratios $\chi(R,R)$ at $\beta = 2.5$.

gauge are in excellent agreement with the usual string tensions (see below), while the agreement in the indirect version is not so good. From these examples it is already apparent that when a global gauge-fixing is imposed, the equality of string tensions extracted from projected and unprojected loops is by no means guaranteed. But a much more striking difference between the gauge-fixed and non-gauge-fixed cases is found at short distances.

Since the abelian and center projected potentials agree *exactly* with the full potential (up to an additive constant) in the absence of gauge-fixing, it follows that at short distances the projected potentials must have a Coulombic form. In contrast, Creutz ratios of the center-projected lattice in maximal center gauge are basically constant, starting with $\chi(2,2)$; this means that the potential is linear starting at one lattice spacing. This "precocious" linearity, for center projection in maximal center gauge, is illustrated in fig. 5, where we compare center projected Creutz ratios at $\beta = 2.5$, to the corresponding full Creutz ratios $\chi(R,R)$ quoted in ref. [15]. A sampling of center projected Creutz ratios in maximal center gauge, compared to the asymptotic string tension reported in ref. [16], is shown at various values of $\beta = 2.3,\ 2.4,\ 2.5$ in fig. 6. This linearity of the center projected potential deep in the Coulombic regime is clearly not just a small perturbation of the non-gauge-fixed result.

The key point here is that, in maximal center gauge, the distribution of P-vortices is precisely that of center vortices, which are large, extended, physical objects. Only this distribution, reflecting long-range, confining physics, contributes to Wilson loops on the center-projected lattice. Short-range fluctuations which, on the unprojected

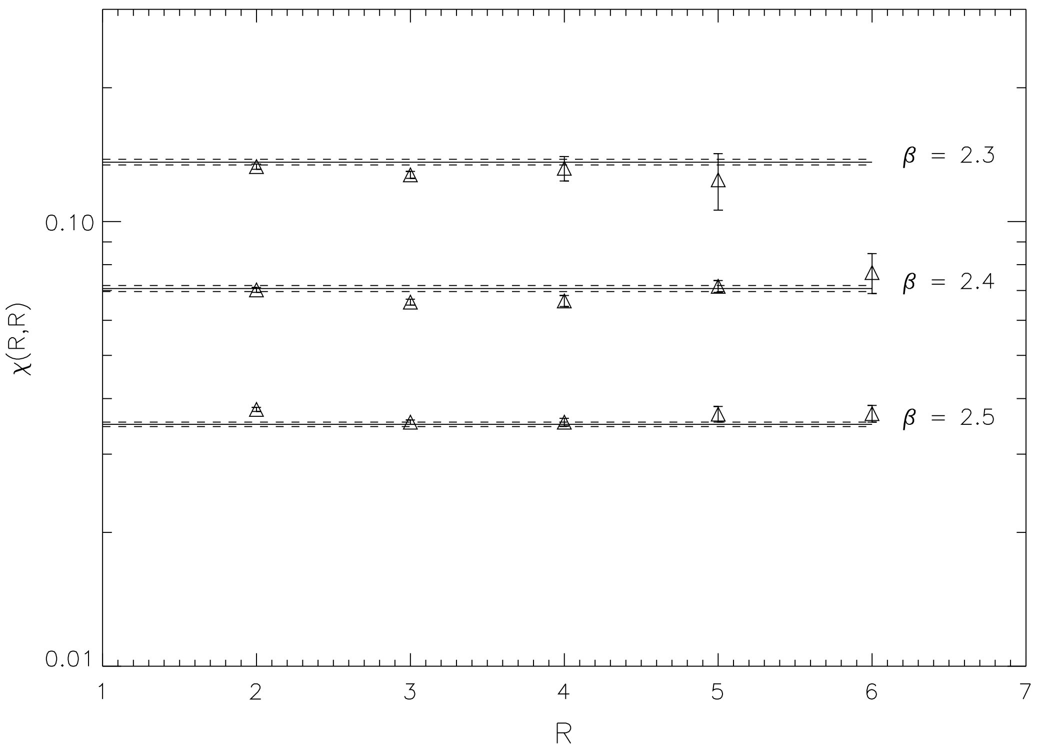

Figure 6: Center-projection Creutz ratios $\chi(R,R)$ vs. R at $\beta = 2.3,\ 2.4,\ 2.5$. Triangles are our data points. The solid lines show the value (at each β) of the asymptotic string tension of the unprojected configurations, and the dashed lines, above and below each solid line, indicate the associated error bars, quoted in ref. [16].

lattice, are responsible for the Coulomb potential, are simply removed by the projection. In contrast, with no gauge fixing, the distribution of P-vortices in some region of scale R just reflects the underlying fluctuations (including gaussian fluctuations) on the unprojected lattice at the scale R, and is only marginally affected, at small scales, by the presence or absence of "thick" center vortices on the unprojected lattice. This is why the projected potential, with no gauge-fixing, only recovers the Coulomb potential at short distances.

A similar effect is found in a variant of abelian projection in maximal abelian gauge, known as "monopole dominance". Monopoles are first located on abelian-projected lattices by the DeGrand-Toussaint criterion, and their contribution to the abelian Wilson loops is computed using the lattice Coulomb propagator. The potential extracted from such "monopole" Wilson loops then displays precocious linearity [13]. Here again, the distribution of monopoles is presumably governed by long-range physics. The magnetic fields assigned to those monopoles by the "monopole-dominance" procedure do not include the high-frequency field fluctuations responsible for the Coulomb potential; there are only large-scale fluctuations resulting in a linear potential.

The relation between monopole and vortex distributions has been discussed in ref. [12], where it was found that monopoles, identified in abelian projection, lie along center vortices, identified in center projection, in monopole-antimonopole chains. The gauge-invariant plaquette action around monopole cubes has also been computed, and it is found that almost all the excess plaquette action, above the vacuum average, lies on plaquettes of the cube pierced by P-vortices. Moreover, this action distribution is almost identical to that of any cube pierced by a P-vortex line, with no monopole inside. This indicates that the monopoles identified in abelian projection are rather undistinguished regions of center vortices, as discussed in ref. [12]. Nevertheless, the fact that center vortices, under abelian projection in maximal abelian gauge, appear as monopole-antimonopole chains, no doubt underlies the approximate agreement of the monopole-dominated and center-projected potentials (both of which display precocious linearity).

5. Conclusions

We have shown that in the absence of gauge-fixing, potentials $V_P(R)$ and $V(R)$ obtained, respectively, from projected and unprojected Wilson loops, agree exactly (up to an additive constant) at all R. On the other hand, such agreement does not hold automatically when global gauge-fixing is imposed, particularly at short distances. In view of this, the criterion that $V_P(R)$ and $V(R)$ have the same asymptotic behavior should be viewed only as a *necessary* condition that configurations identified on projected lattices are physical objects, with the required confinement properties. Abelian and center dominance in themselves are by no means a sufficient indicator of the physical nature of monopoles and/or vortices identified on projected lattices.

There are, however, a number of other tests which can be used to establish the physical nature of topological objects identified on projected lattices. In the case of center vortices, where the vortices are identified by projection in maximal center gauge, it was found that:

1. if a Wilson loop is linked to a region which, according to the center-projection, contains a vortex, then that Wilson loop picks up a relative minus sign ($W_n/W_0 \to (-1)^n$);

2. the presence of vortices in the projected lattice is correlated to the existence of a string tension on the unprojected lattice ($\chi_0(R,R) \to 0$);

3. P-vortex densities scale as required by asymptotic freedom, in a way which is appropriate to a density of surfaces;

4. there is a very large excess plaquette action at plaquettes pierced by P-vortices.

Properties 1, 2, and 4 involve correlations between the P-vortices of the projected lattice with gauge-invariant quantities (Wilson loops, Creutz ratios, and plaquette actions, respectively) on the unprojected lattice, while property 3 is required if P-vortices on the center-projected lattice correlate with the location of physical, surface-like objects on the unprojected lattice.

We have shown in this paper that all of the criteria 1–4 above, for the physical nature of vortices located via center projection, fail completely if no gauge-fixing is imposed. This is a simple consequence of the fact that vortices are identified using local operators (projected plaquettes), and in the absence of a global gauge-fixing these operators have no information about physics on scales much larger than one lattice spacing. The non-triviality of center projection, and its ability to locate confining center vortex configurations, can be attributed entirely to the effect of fixing to maximal center gauge. This does not mean that the center vortex mechanism is in any sense gauge-dependent, and in fact much of the data shown above concerns the effect of vortices on gauge-invariant observables. It is important to distinguish between the procedure for locating vortices, and the center vortices themselves; it is only our method for *finding* vortices which relies on a gauge choice.

Finally, the fact that the center-projected potential has the correct string tension remains an important property of maximal center gauge. It is true that this property is a triviality in the absence of gauge-fixing, as shown in ref. [2] and also here. However, as noted in the last section, the agreement of projected and standard string tensions is neither trivial nor inevitable when a global gauge-fixing is imposed and "precocious linearity" is obtained; this agreement is needed in order to establish that fluctuations of center vortices alone lead to the correct value of the string tension. It is the tests of physicality 1–4 above, *combined* with the property of center dominance in maximal center gauge, that together make the case for center vortices as the quark confinement mechanism.

Acknowledgments

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A. Appendix

In this appendix we show why $W_{even}(C) \approx W_{odd}(C) \approx W(C)$, in the absence of gauge fixing, up to very large distance scales. If we denote by

$$Z(C) = \prod_{l \in C} \mathrm{signTr}[U_l], \tag{A.1}$$

then it is not hard to see that

$$\begin{aligned}
W_{even}(C) &= \frac{\left\langle \frac{1}{2}(1+Z(C))\frac{1}{2}\mathrm{Tr}[U(C)]\right\rangle}{\left\langle \frac{1}{2}(1+Z(C))\right\rangle}\,, \\
W_{odd}(C) &= \frac{\left\langle \frac{1}{2}(1-Z(C))\frac{1}{2}\mathrm{Tr}[U(C)]\right\rangle}{\left\langle \frac{1}{2}(1-Z(C))\right\rangle}\,,
\end{aligned} \tag{A.2}$$

Applying eq. (2.15), and also using the fact that

$$\chi_{1/2}[U(C)]^2 = 1 + \chi_1[U(C)]\,, \tag{A.3}$$

we have

$$\begin{aligned}
W_{even}(C) &\approx W(C) + 2\left(\frac{4}{3\pi}\right)^{P(C)}\,, \\
W_{odd}(C) &\approx W(C) - 2\left(\frac{4}{3\pi}\right)^{P(C)}\,,
\end{aligned} \tag{A.4}$$

Since $W(C)$ has an area-law falloff, it is clear that when the area of the loop is much larger than the perimeter, the first term on the rhs, $W(C)$, can be neglected in comparison to the second term. That would give $W_{odd}/W_{even} \to -1$, which is in apparent contradiction to the Monte Carlo results shown in Fig. 1. The paradox is resolved by computing how large the loop C has to be, for this limiting ratio to be obtained. Consider for simplicity, square Wilson loops, and let L be the length of a side such that

$$W(L,L) = \left(\frac{4}{3\pi}\right)^{4L}\,. \tag{A.5}$$

For loops of somewhat smaller area, we can neglect the second, rightmost terms in eq. (A.4), so that $W_{even} \approx W_{odd}$.[3] From

$$W(L,L) = \exp\left[-\sigma L^2 - 4bg^2(a)L\right] \tag{A.6}$$

we have

$$\begin{aligned}
L &= \frac{4}{\sigma}\Big(\log\frac{3\pi}{4} - bg^2(a)\Big) \\
&\approx \frac{4}{\sigma}\log\frac{3\pi}{4}\,,
\end{aligned} \tag{A.7}$$

where we have used the fact that $g^2(a) \to 0$ in the continuum, $\beta \to \infty$ limit. Since $\sigma(\beta) \to 0$ as $\beta \to \infty$, it is clear that $L \to \infty$ in lattice units in the same limit. Convert now to physical units

$$L_p = La\,, \qquad \sigma_p = \frac{\sigma}{a^2} \tag{A.8}$$

[3]For example, in the case of $\beta = 2.3$, we find $W(5,5) \approx 4\times 10^{-4}$, while for this loop the second term $2(4/3\pi)^{P(C)}$ in eq. (A.4) is approximately 7×10^{-8}.

and we find

$$L_p = \frac{4\sigma_p^{-1}\log\frac{3\pi}{4}}{a}. \tag{A.9}$$

Since $a(\beta) \to 0$ goes exponentially to zero as $\beta \to \infty$, it follows that L_p diverges to infinity in *physical* units. The conclusion is that, in the continuum limit,

$$W_{even}(C) = W_{odd}(C) = W(C)\,, \tag{A.10}$$

for all loops C, and, as a consequence,

$$\chi_{even}(I,J) = \chi(I,J) \tag{A.11}$$

everywhere, in the absence of maximal center gauge-fixing. This explains the results shown above in figs. 1 and 2.

References

[1] J. Ambjørn and J. Greensite, *J. High Energy Phys.* **05** (1998) 004 [hep-lat/9804022].

[2] M. Ogilvie, hep-lat/9806018.

[3] L. Del Debbio, M. Faber, J. Giedt, J. Greensite, and Š. Olejník, *Phys. Rev.* **D 58** (1998) 094501 [hep-lat/9801027].

[4] T. Kovács and E. Tomboulis, *Phys. Rev.* **D 57** (1998) 4054 [hep-lat/9711009].

[5] P.W. Stephenson, *Nucl. Phys.* **B 539** (1999) 577 [hep-lat/9807019].

[6] M. Göpfert, *Nucl. Phys.* **B 190** (1981) 151.

[7] L. Susskind in *Les Houches 1976: Weak and electromagnetic interactions at high energies*, R. Balian and C.H. Llewellyn Smith eds., North-Holland, Amsterdam 1977.

[8] J. Ambjørn, P. Olesen, and C. Peterson, *Nucl. Phys.* **B 240** (1984) 198; *Nucl. Phys.* **B 240** (1984) 533;
C. Michael, *Nucl. Phys.* **26** *(Proc. Suppl.)* (1992) 417; *Nucl. Phys.* **B 259** (1985) 58;
N. Cambell, I.H. Jorysz, and C. Michael, *Phys. Lett.* **B 167** (1986) 91;
M. Faber and H. Markum, *Nucl. Phys.* **4** *(Proc. Suppl.)* (1988) 204;
M. Muller, W. Beirl, M. Faber, and H. Markum, *Nucl. Phys.* **26** *(Proc. Suppl.)* (1992) 423;
G. Poulis and H. Trottier, *Phys. Lett.* **B 400** (1997) 358 [hep-lat/9504015].

[9] O. Phillipsen and H. Wittig, *Phys. Rev. Lett.* **81** (1998) 4056 [hep-lat/9807020];
F. Knechtli and R. Sommer, *Phys. Lett.* **B 440** (1998) 345 [hep-lat/9807022];
C. Stewart and R. Koniuk, hep-lat/9811012;
H. Trottier, hep-lat/9812021.

[10] T. Suzuki and Y. Yotsuyanagi, *Phys. Rev.* **D 42** (1990) 4257.

[11] K. Langfeld, H. Reinhardt, and O. Tennert, *Phys. Lett.* **B 419** (1998) 317 [hep-lat/9710068].

[12] L. Del Debbio, M. Faber, J. Greensite, and Š. Olejník, in *New developments in quantum field theory*, P. Damgaard and J. Jurkiewicz eds., Plenum, New York 1998 [hep-lat/9708023].

[13] G.S. Bali, V. Bornyakov, M. Mueller-Preussker, and K. Schilling, *Phys. Rev.* **D 54** (1996) 2863 [hep-lat/9603012].

[14] L. Del Debbio, M. Faber, J. Greensite, and Š. Olejník, *Phys. Rev.* **D 55** (1997) 2298 [hep-lat/9610005].

[15] F. Gutbrod, *Z. Physik* **C 37** (1987) 143.

[16] G.S. Bali, C. Schlichter, and K. Schilling, *Phys. Rev.* **D 51** (1995) 5165 [hep-lat/9409005].